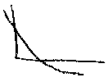
2a) $x = r_1 \cdot r_2 \cdot r_3$ periphere substitutionale Fkt.

$1 = 1 \cdot 1 \cdot 1$ kein Faktor darf ∅ werden

$1 = 0{,}5 \cdot 2 \cdot 1$

c) $x = 2r_1$ limitational also nicht substitutionierbar

d) $x = r_1 + r_2$ total substitutionierbar



e) $x = r_1 \cdot r_2 + r_1$ peripher subst.

b) $\begin{Bmatrix} x = r_1 \\ x = 2r_2 \end{Bmatrix}$ limitational linear da $\Rightarrow \frac{r_1}{r_2} = \frac{x}{\frac{x}{2}} = 2 = \text{const.}$

$$\left.\begin{array}{l} \frac{r_1}{r_j} = \text{const.} \\ a_i = \frac{r_i}{x} = \text{const.} \end{array}\right\} \text{linear} \qquad \left.\begin{array}{l} a_1 = \frac{r_1}{x} = \frac{x}{x} = 1 \\ a_2 = \frac{r_2}{x} = \frac{\frac{x}{2}}{x} = \frac{1}{2} \end{array}\right\} \text{const.}$$

f) $\begin{Bmatrix} x = r_1^2 \\ x = r_2^2 \end{Bmatrix}$ $\frac{r_1}{r_2} = \frac{\sqrt{x}}{\sqrt{x}} = 1 = \text{const.}$

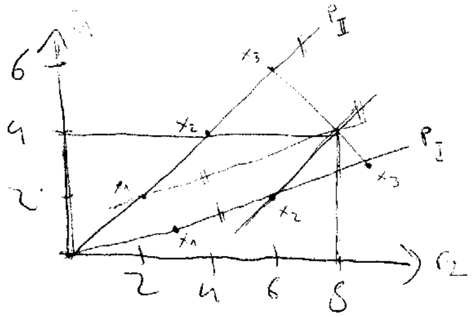
nicht linear weil:

$$\left.\begin{array}{l} a_1 = \frac{r_1}{x} = \frac{\sqrt{x}}{x} \neq \text{const.} \\ a_2 = \frac{r_2}{x} = \frac{\sqrt{x}}{x} \neq \text{const.} \end{array}\right\} \text{nicht linear limitational}$$

3a) $a_{1I} = 1$ $a_{2I} = 3$

$a_{1II} = 2$ $a_{2II} = 2$

$\left.\begin{array}{l} r_1 = 4 \\ r_2 = 8 \end{array}\right\} x = ?$



P_I: $r_1 = 2$ $r_2 = 6$ $\Rightarrow x = 2$

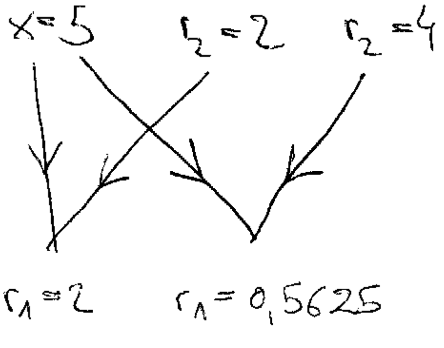
P_{II}: $r_1 = 2$ $r_2 = 2$ $\Rightarrow x = 1$

4 8 $\Rightarrow$ 3

$P_I = \begin{Bmatrix} x = r_1 \\ x = \frac{1}{3} r_2 \end{Bmatrix}$

$P_{II} = \begin{Bmatrix} x = \frac{1}{2} r_1 \\ x = \frac{1}{2} r_2 \end{Bmatrix}$

1.) $x = \frac{1}{2} r_2 + 2 \sqrt{r_1 \cdot r_2}$



$$\frac{x - 0{,}5 r_2}{2} = \sqrt{r_1 \cdot r_2}$$

$$r_1 = \frac{\cancel{r_2}\ 1}{4 r_2} \left(x^2 - \underset{2}{r_{\cancel{1}} x} + \frac{1}{4} r_2^2 \right)$$

$$RTS_{12} = \left| \frac{dr_1}{dr_2} \right|$$

$$RTS_{12} = \left| -\frac{6{,}25}{r_2^2} + 0{,}0625 \right|$$

$x = 5$

$$r_1 = \frac{6{,}25}{r_2} + 0{,}0625\, r_2 - 1{,}25$$

$r_2 = 2$

$RTS_{12} = 1{,}5$ $RTS_{12} = 0{,}3285$

$(r_2 = 2;\ x = 5)$ $(r_2 = 4;\ x = 5)$

$X = \frac{1}{2} r_2 + 2 \sqrt{r_1 r_2}$ $r_2 = 4$ $r_1 \rightarrow X$

$x = 2 + 4 \sqrt{r_1}$

$MP_i = \frac{dx}{dr_i}$ $AP_i = \frac{x - \bar{x}}{r_i}$

Bsp.: 6

a) $x_0 = 2 \cdot r_1 r_2$ $\quad x = 2 \cdot (\mu r_1)(\mu r_2) = \mu^2 \cdot 2 r_1 r_2 = \mu^2 \cdot x_0$

Homogenitätsgrad $c = 2$

$c > 1 \rightarrow$ doppelte Input $\Rightarrow$ mehr als 2fach OP

$c < 1 \rightarrow$ —— " —— $\Rightarrow$ weniger als 2fach

$c = 1 \rightarrow$ linear

b) $x_0 = 3 r_1^2 + 2 r_1 r_2$ $\quad x = 3(\mu r_1)^2 + 2(\mu r_1)(\mu r_2) = \mu^2 \cdot x_0 \Rightarrow c = 2$

c) $x_0 = 5 r_1 + 5 r_1 r_2$ $\quad x = 5 \mu r_1 + 5 \mu^2 r_1 r_2 \longrightarrow$ inhomogen

d) $a + b = c$

e) $c = 1$

f) inhomogen

Bsp 7) $x = 2 r_1^2 + 3 r_2^2 + 10 r_1 r_2 \quad \Rightarrow c = 2$

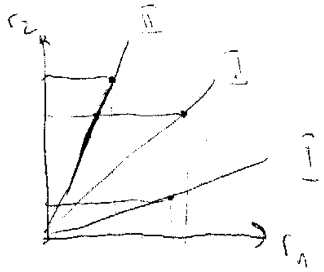
total Substitutional

Durchschnittsproduktivität

$$AP_1 = \frac{x - \bar{x}}{r_1} = \frac{2 r_1^2 + \cancel{3 r_2^2} + 10 r_1 r_2 - \cancel{3 r_2^2}}{r_1} = 2 r_1 + 10 r_2$$

Grenzproduktivität

$$MP_1 = \frac{dx}{dr_1} = 4 r_1 + 10 r_2$$



II gar schlecht

Kosten

$K = r_1 \cdot q_1 + r_2 \cdot q_2 + \ldots$ Menge x Preis

! min !

Bsp ~~8~~ 9) 2

$x = 2r_1^2 + 7r_2^2 + 15r_1r_2$ $q_1 = 6$ $q_2 = 10$ $x = 100$

a) $\Pi P_1 = \frac{dx}{dr_1} = 4r_1 + 15r_2$

$\Pi P_2 = 14r_2 + 15r_1$

$AP_1 = 2r_1 + 15r_2$

$AP_2 = 7r_2 + 15r_1$

b) $K = r_1 \cdot q_1 + r_2 \cdot q_2 = 6r_1 + 10r_2$ min!

$100 = 2r_1^2 + 7r_2^2 + 15r_1r_2$

$L = 6r_1 + 10r_2 + \lambda(2r_1^2 + 7r_2^2 + 15r_1r_2 - 100)$

1) $\frac{dL}{dr_1} = 6 + \lambda(4r_1 + 15r_2) = 0$

2) $\frac{dL}{dr_2} = 10 + \lambda(14r_2 + 15r_1) = 0$

3) $\frac{dL}{d\lambda} = 2r_1^2 + 7r_2^2 + 15r_1r_2 - 100 = 0$

1) $\Rightarrow \frac{6}{4r_1 + 15r_2} = -\lambda$

2) $\Rightarrow \frac{10}{14r_2 + 15r_2} = -\lambda$

1) = 2) $84r_2 + 30r_1 = 40r_1 + 150r_2 \mathrel{\hat{=}} q_1 \cdot \Pi P_2 = q_2 \cdot \Pi P_1 = \frac{q_1}{q_2} = \frac{\Pi P_1}{\Pi P_2}$

$r_1 = \frac{66}{50} r_2 = 1{,}32 r_2$ ⇒ in 3) einsetzen

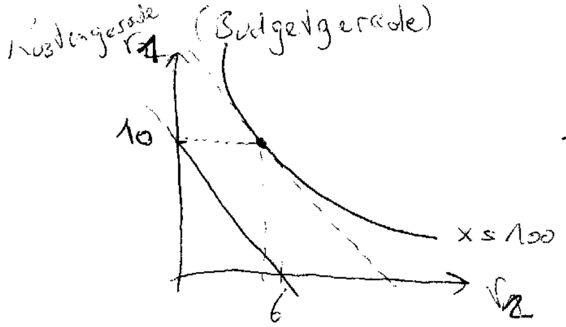
$2 \cdot (1{,}32 r_2)^2 + 7r_2^2 + 15 \cdot (1{,}32 r_2^2) = 100$

$r_2 = 1{,}817$

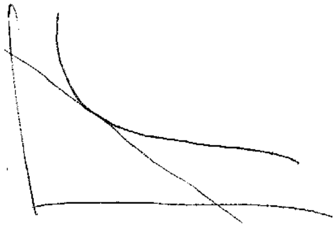
$r_1 = 2{,}4$

$K(x=100) = 6 \cdot 2{,}4 + 10 \cdot 1{,}817 = 32{,}56$

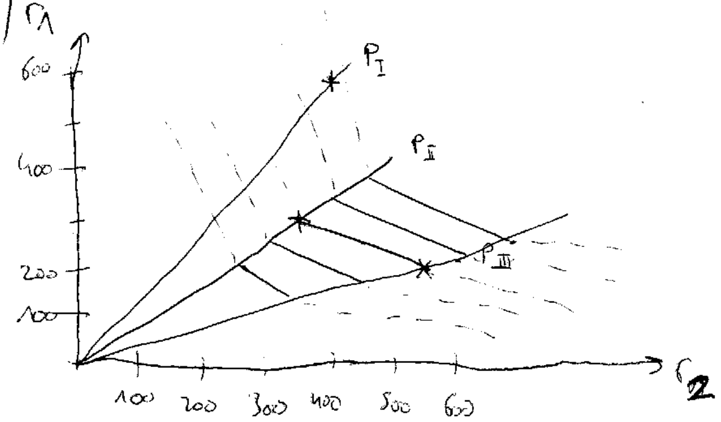
$k = \frac{K}{} = 0{,}3256$ GE



Steigung der Gerade $\frac{q_1}{q_2} = \frac{\Pi P_1}{\Pi P_2}$



10.)



$$300 = 350k + d$$
$$200 = 550k + d$$
$$100 = -200k \Rightarrow k = 0,5$$
$$d = 475$$

$r_1 = 475 - 0,5\, r_2$

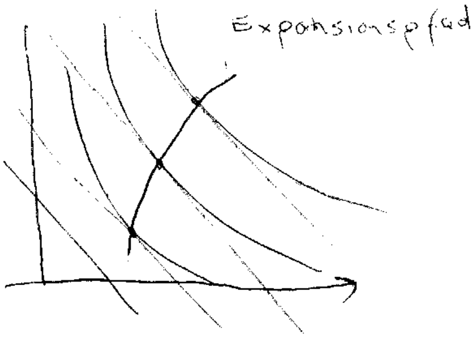
nur für $200 \leq r_1 \leq 300$
$350 \leq r_2 \leq 550$

Grenzrate der Substitution $RTS_{1,2} = \left| \frac{dr_1}{dr_2} \right| = \underline{\underline{\frac{1}{2}}}$

Bsp.) 12) $x = \frac{1}{6}\sqrt{r}$ $q = 36$

$K = 36 \cdot r = 36 \cdot 36x^2 = (36x)^2$ ... Gleichung d. Expansionspfad

13.)



$x = r_1^{0,4} \cdot r_2^{0,4}$ $\quad q_1 = 2 \quad q_2 = 3 \quad$ K(x)?

$$\frac{q_1}{q_2} = \frac{MP_1}{MP_2} \Rightarrow \frac{2}{3} = \frac{\cancel{0,4} \cdot r_1^{-0,6} \cdot r_2^{0,4}}{\cancel{0,4} \cdot r_1^{0,4} \cdot r_2^{-0,6}} \Rightarrow \frac{2}{3} = \frac{r_2}{r_1}$$

$$2r_1 = 3r_2$$

$$r_1 = \frac{3}{2} r_2$$

$$x = \left(\frac{3}{2} r_2\right)^{0,4} \cdot r_2^{0,4} = \left(\frac{3}{2}\right)^{0,4} \cdot r_2^{0,8}$$

$$r_2 = x^{1,25} \cdot \left(\frac{3}{2}\right)^{-0,5} \quad ?{:}0,4$$

$$r_1 = x^{1,25} \cdot \left(\frac{3}{2}\right)^{-0,5} \cdot \frac{3}{2} = x^{1,25} \cdot \left(\frac{3}{2}\right)^{0,5}$$

$$K = r_1 \cdot q_1 + r_2 \cdot q_2 = 2 \cdot x^{1,25} \cdot \left(\frac{3}{2}\right)^{0,5} + 3 \cdot x^{1,26} \cdot \left(\frac{3}{2}\right)^{-0,5} = x^{1,25}\left(2 \cdot \sqrt{\frac{3}{2}} + 3\sqrt{\frac{2}{3}}\right) =$$

$$= \sqrt{24} \cdot x^{1,25}$$

25.3.98

14) $K = 12000 + 60x \qquad 0 \leq x \leq 600 \qquad$ Marktpreis 90GE

$$G = E - K = 90x - 12000 - 60x = 30x - 12000$$

$$G = 0 \Rightarrow x_{BE} = 400$$

Optimum $G' = 0 \Leftrightarrow E' = K'$

? $90 = 60$?

$$x_{max} = 600$$

$$k(x) = \frac{K(x)}{x} = \frac{12000}{x} + 60$$

$$k'(x) = 0 \Rightarrow x_{opt} = 600$$

15.) $x = 4 \cdot r_1 \cdot r_2$ $\quad q_1 = 1$ $\quad p = 10$

$q_2 = 3$

a) $5 = 4 \cdot r_1 \cdot r_2 \Rightarrow r_1 = \frac{5}{4 r_2}$

b) $\frac{q_1}{q_2} = \frac{MP_1}{MP_2} \qquad \frac{1}{3} = \frac{4 r_2}{4 r_1} \Rightarrow r_1 = 3 r_2$

$x = 4 \cdot 3 r_2 \cdot r_2 = 12 r_2^2 \Rightarrow r_2 = \sqrt{\frac{x}{12}} \qquad r_1 = 3\sqrt{\frac{x}{12}}$

c) $K = r_1 q_1 + r_2 q_2$

$K(x) = 3\sqrt{\frac{x}{12}} + 3\sqrt{\frac{x}{12}} = 6\sqrt{\frac{x}{12}} = \sqrt{3x}$

d) $G = E - K = p \cdot x - \sqrt{3x} = 10x - \sqrt{3x}$

$G' = 0 = 10 - \frac{1}{2}\sqrt{3}\,\frac{1}{\sqrt{x}} = 0$

$x = 0,00075$

$G'' > 0 \Rightarrow$ Min!

$G = 0 \Rightarrow E = K$

$10x = \sqrt{3x} \Rightarrow x_{BE} = 0,03$

1.) $K = x^3 - 24x^2 + 197x + 14$ $\qquad p = 80$

$G = 0 \Rightarrow E = K$

$G = -x^3 + 24x^2 - 117x - 14 = 0$

$x_1 = 7$

$x_2 = 17,11 \qquad x_{BE} = 7$

~~$x_3 = -0,11$~~

$G' = 0 = 3x^2 - 48x + 117 = 0$

$x^2 - 16x + 39 = 0$

$G'' = \cancel{2x - 16}$

$-6x + 48$

$G(3) = -320$

$x_1 = 3 \Rightarrow G''(3) > 0 \Rightarrow$ Mi

$x_2 = 13 \Rightarrow G''(13) < 0 \Rightarrow$ Max

$G(13) = 324$

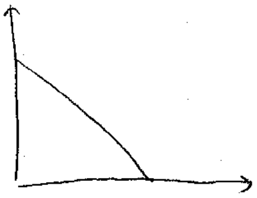
9 $k(x) = x^2 - 24x + 197 + \frac{14}{x} \Rightarrow 2x - 24 - \frac{14}{x^2} = 0$

$x_{Betriebs,opt} = 12,05$

$k_v = x^2 - 24x + 197$

$2x - 24 \Rightarrow x = 12$ ... $x_{Betriebsmin}$

17.) $p = 300 - 0,02x$



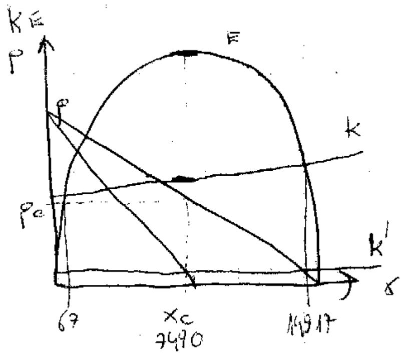
$\Rightarrow E = p \cdot x = p(x) \cdot x = 300x - 0,02x^2$

$k = 20000 + 0,4x$

$G = E - K \Leftrightarrow E' = K'$

$300 - 0,04x = 0,4$

$x_{Cournot} = 7490$ $p = 300 - 7490 \cdot 0,02 = 150,2$



$G = 7490 \cdot 150,2 - 20000 - 7490 \cdot 0,4 = 1,1$ Mio.

$G = 0 \Rightarrow E = K$

$300x - 0,02x^2 = 20000 + 0,4x$

$x^2 - 14980x + 1$ Mio $= 0$

$x_{1,2}$ $\begin{cases} 67 \\ 14912 \end{cases}$

$x = 4000 \Rightarrow p = 220$ $G = 4000 \cdot 220 - 20000 \cdot 0,4 \cdot 4000 = 858400$

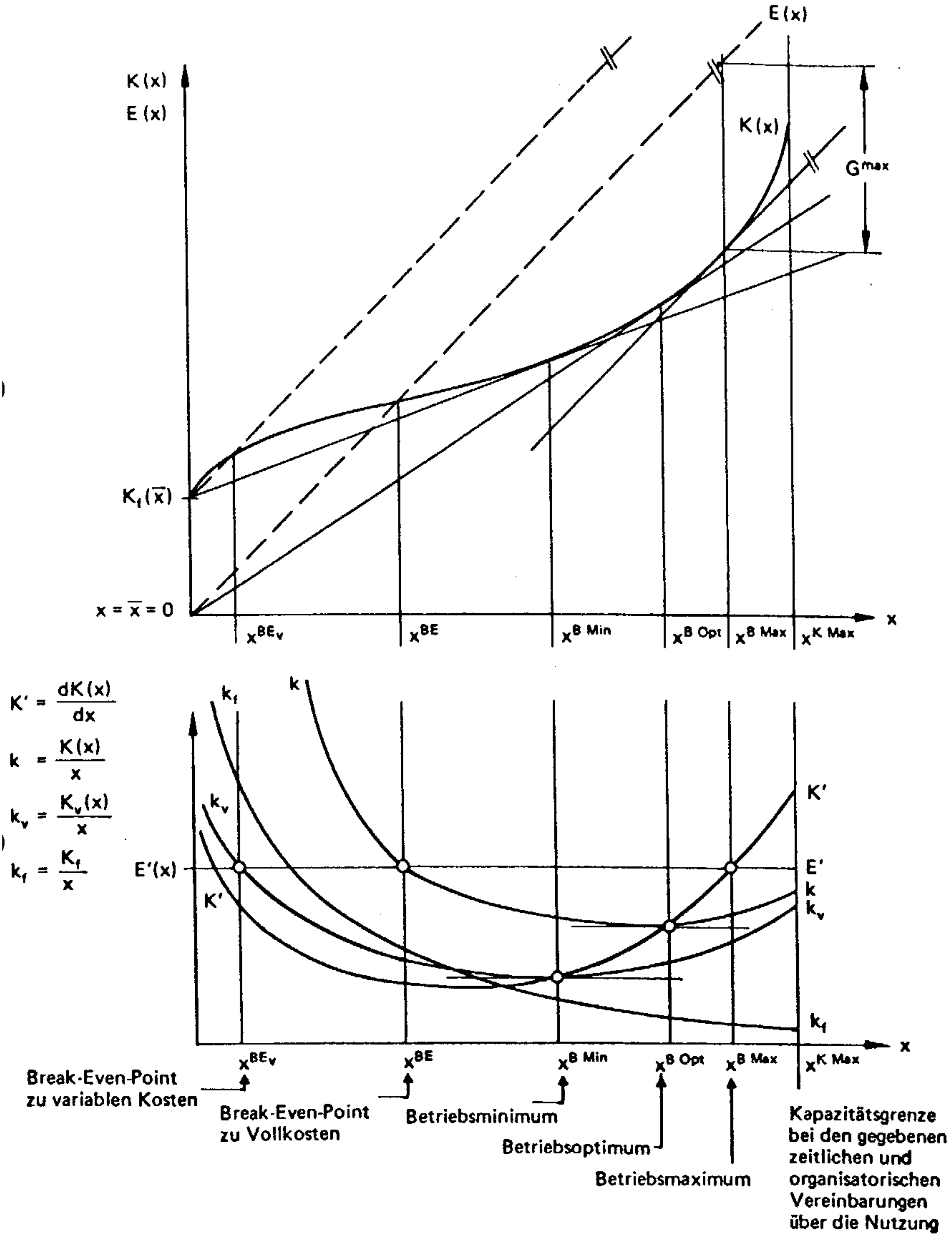


Abb. 1.17. Kosten- und Erlösfunktion und Derivate der Kosten- und Erlösfunktion

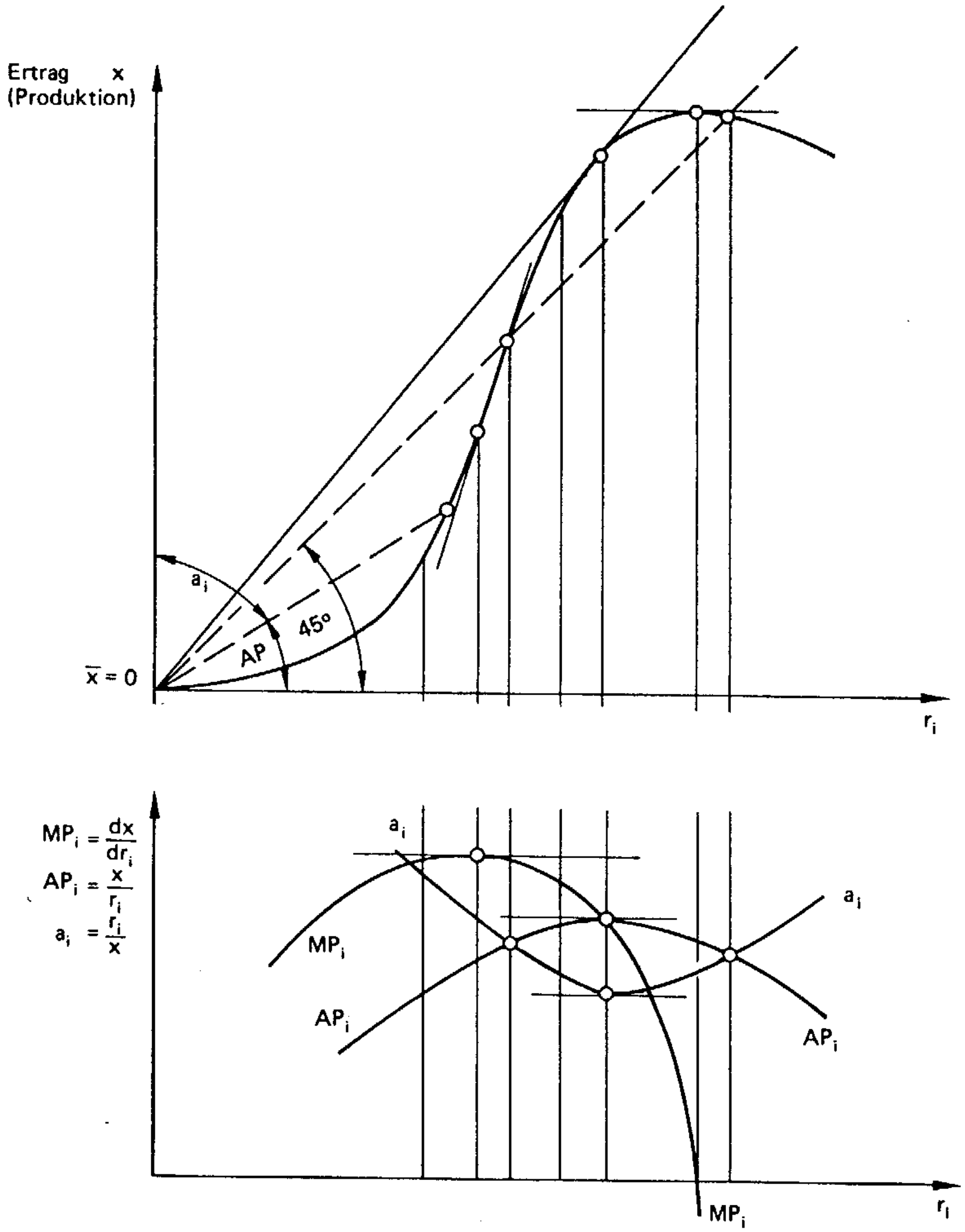


Abb. 1.16. Produktionsfunktion vom Typ A bei partieller Variation des Faktors r_i und Derivate ($\bar{x} = O$ bedeutet, daß keine Produktion ohne r_i möglich ist)

18) $I: p = 800 \quad x = 1400 \quad k = 960.000$

$II: p = 920 \quad x = 1250 \quad k = 900000$

$p = p_0 - bx$

$k = k_F + k_v \cdot x$

$800 = p_0 - 1400b$

$\underline{920 = p_0 - 1250b}$

$120 = 150b \Rightarrow b = 0{,}8 \Rightarrow p_0 = 1920$

$p = 1920 - 0{,}8x$

$960000 = K_F + k_v \cdot 1400$

$\underline{900000 = k_F + k_v \cdot 1250}$

$60000 = k_v \cdot 150 \Rightarrow k_v = 400 \Rightarrow k_F = 400000$

$k = 400000 + 400x$

$E = p \cdot x$

$G = E - K = 1920x - 0{,}8x^2 - 400000 - 400x$ (Gewinnoptimierung)

$G' = 0 \Rightarrow 1920 - 1{,}6x - 400 = 0 \Rightarrow x_c = 950 \Rightarrow p_c = 1160$

$G_c = 1102000 - 780000 = \underline{\underline{322000}}$

$E \Rightarrow \max!$ (Umsatzoptimierung)

$E' = 1920 - 1{,}6x = 0 \quad x = 1200 \Rightarrow p = 960 \Rightarrow E = 1152000$

$\underline{K = 880.000}$

$G = 272000$

19) $G = 145000$

20.) $K = 5000 + 20x$

$p = 200 - x$

$E' = K'$

$200 - 2x = 20$

$x_c = 90 \Rightarrow p_c = 110 \Rightarrow G_c = 9900 - 6800 = 3100$

$\downarrow +2500$

b) neues K $K = 2500 + 20x$

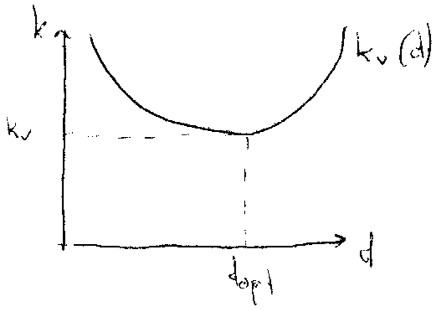
$x_c = 90 \quad p_c = 110 \quad G_c = 9900 - 4300 = 5600$

c) $K = 5000 + 10x$

$200 - 2x = 10$

$x_c = 95 \quad p_c = 105 \quad G_c = 9975 - 5950 = 4025$

Intensität d



$x = d \cdot \overset{\text{Zeit}}{T} \cdot (n)$

v... für 1 Output

$k_v(d) = \sum_{i=1} v_i(d) \cdot q_i$

22) $v(d) = 2\cdot(d-3)^2 - 3\cdot d + 15$ $\quad 1 \le d \le 10$

b) $k_v(d) = 10(d-3)^2 - 15d + 75$

$k_v(d) = 20(d-3) - 15 = 0 \Rightarrow d_{opt} = 3{,}75$

$k_v(3{,}75) = 24{,}375$

$K(x) = 24{,}375 \cdot x + K_f = 24{,}375 \cdot 3{,}75 \cdot T$

a) $K = k_v(d)\cdot x = v(d)\cdot q \cdot x = \left[10(d-3)^2 - 15d + 75\right]\cdot x =$

$d = \frac{x}{8}$ $\quad = \frac{10}{64}x^3 - \frac{75}{8}x^2 + 165x$

$1 \le d \le 10 \quad t = 8$

$8 \le x \le 80$

Intensitätssplitting

$d_{min} \quad t_{min} \qquad d_{sp} \cdot t_{sp} \qquad t_{min} + t_{sp} = T$

24k)

$$\min\left[\frac{k_v(d_{sp})\cdot d_{sp} - k_v(d_{min})\cdot d_{min}}{d_{sp} - d_{min}}\right] = \frac{(152 - 2d_{sp} + 0{,}01d_{sp}^2)\cdot d_{sp} - 88\cdot 40}{d_{sp} - 40} =$$

$$= \frac{0{,}01d_{sp}^3 - 2d_{sp}^2 - 152d_{sp} - 3520}{d_{sp} - 40} = \min[\;] = 0{,}02d_{sp} - 16 = 0 \Rightarrow d_{sp} = 80$$

$0{,}01d_{sp} - 2d_{sp}^2 - 152d_{sp} - 3520 : (d_{sp} - 40) = 0{,}01d_{sp}^2 - 1{,}6d_{sp} + 88$

$-\underline{0{,}01d_{sp} + 0{,}4d_{sp}^2}$

$-1{,}6d_{sp}^2$

$-\underline{1{,}6d_{sp}^2 + 64d_{sp}}$

$88d_{sp}$

$-\underline{88d_{sp} + 3520}$

$t_{min} + t_{sp} = t \Rightarrow t_{sp} = 200 - t_{min}$

$x = d \cdot t$

$x = d_{min} \cdot t_{min} + d_{sp} \cdot t_{sp}$

$15\,000 = — \quad — \quad ——$

$15000 = 40 \cdot t_{min} + 80\, t_{sp}$

$15\,000 = 40\, t_{min} + 80 \left(200 - t_{min}\right) \Rightarrow t_{min} = 25$

$t_{sp} = 175$

$K = K_F + k_v(d) \cdot x = K_F + x_{sp} \cdot k_v\left(d_{sp}\right) + x_{min} \cdot k_v\left(d_{min}\right) =$

$= 50000 + 80 \cdot 175 \cdot 56 + 40 \cdot 25 \cdot 88 = 922.000$

< 923.750

$x_{min} = d_{min} \cdot t_{min} = 40 \cdot 25$

$x_{sp} = 80 \cdot 175$

26) a) $v(d) = 0{,}0001\, d^2 - 0{,}008 \cdot d + 0{,}22$

$10 \leq d \leq 80$

$x = 200$

$t = 8$

$v'(d) = 0 = 0{,}0001 d - 0{,}008 \Rightarrow d_{opt} = 40$

$v(40) = 0{,}06\, l/km$

$V = v(d_{opt}) \cdot x = 12l$

$t = \frac{x}{d} = \frac{200}{40} = 5h$

b) $d = \frac{x}{t} = \frac{200}{8} = 25 km/h \qquad v(25) = 0{,}00825 \qquad V = 16{,}5L \quad t = 8$

$\min \left[\frac{v(d_{sp})\, d_{sp} - v(d_{min})\, d_{min}}{d_{sp} - d_{min}} \right] = 0{,}0002 d_{sp} - 0{,}007 = 0 \Rightarrow d_{sp} = 35$

$t = t_{min} + t_{sp}$

$t_{min} = 8 - t_{sp}$

$x = x_{min} + x_{sp} = d_{min} \left(8 - t_{sp}\right) + d_{sp} \cdot t_{sp}$

$200 = 10 \left(8 - t_{sp}\right) + 35\, t_{sp} \Rightarrow t_{sp} = 4{,}8\, h$

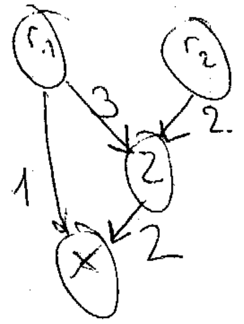
$t_{min} = 3{,}2\, h$

$$V = v(d_{sp}) \cdot x_{sp} + v(d_{min}) \cdot x_{min} = 0{,}0625 \cdot 168 + 0{,}15 \cdot 32 = 15{,}31$$

$\overline{22.4.98}$

27) siehe Skript

28)



$$\begin{aligned} r_1 &= 3 \cdot z + x && + b_{r_1} \\ r_2 &= 2 \cdot z && + b_{r_2} \\ z &= 2 \cdot x && + b_z \\ x &= \emptyset && \underbrace{+ b_x}_{\text{Direktbedarf}} \end{aligned}$$

$$A = \begin{pmatrix} 0 & 0 & 3 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Einsetzen

$z = 2b_x + b_z$

$r_2 = 4b_x + 2b_z + b_{r_2}$

$r_1 = 6b_x + 3b_z + b_x + b_{r_1}$

$$G = \begin{pmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = T^{-1}$$

$$T = E - A = \begin{pmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$G \cdot \begin{pmatrix} 0 \\ 0 \\ 10\,000 \\ 30\,000 \end{pmatrix} = \begin{pmatrix} 240\,000 \cdot r_1 \\ 140\,000 \cdot r_2 \\ 70000 \cdot z \\ 30000 \cdot x \end{pmatrix}$$

$$d_z = \frac{x}{t} = \frac{70\,000}{1000} = 70 \qquad k_{v,z} = \underline{\underline{10}}$$

$$k_v(d_x) = 190 - 8d_x + 0{,}1 d_x^2$$

$$k_v' = -8 + 0{,}2 d_x = \emptyset \Rightarrow d_{x_{opt}} = 40$$

$$k_v(40) = \underline{\underline{30}} = k_{v,x}$$

$$(10, 15, 10, 30) \cdot 6 = (10, 15, \underset{\substack{\uparrow \\ 7}}{70}, \underset{\substack{\uparrow \\ x}}{180})$$

[illegible]

steht eh im Skriptum

29) $q_{\cancel{B}} = \sqrt{\frac{2 \cdot 100 \cdot 260}{26}} \approx 45$ Stk.

$m = \frac{26 \cdot 10}{236} = 11$ Stk.

$K = 100 \cdot \frac{260}{45} + 26 \cdot \frac{45}{2} + 260 \cdot 104 = \underline{\underline{28203}}$

30) $q = \sqrt{\frac{2 \cdot 1040 \cdot 150}{5}} \approx 250$ Stk.

$m = \frac{1040 \cdot 19}{365} \approx 40$ Stk.

$K = 150 \cdot \frac{1040}{250} + 5 \cdot \frac{250}{2} + 1040 \cdot 20 = 22049$

21.4.

$$KW_0 = \sum_{t=0}^{ND} q_t (1+r)^{-t}$$

34)

t_0	t_1	t_2	t_3
-80	20	40	50

$r = 10\%$

$KW_0 = -80 + 20 \cdot (1{,}1)^{-1} + 40 \cdot 1{,}1^{-2} + 50 \cdot 1{,}1^{-3} = 8805$

$KW_3 = -80 \cdot 1{,}1^{3} + 20 \cdot 1{,}1^{2} + 40 \cdot 1{,}1 + 50 = 11720 = 8805 \cdot 1{,}1^{3}$

	t_0	t_1	t_2	t_3
F:	-80	88	96 8	106,48
R:	-80	20	40	50

$20 \rightarrow 22$

$$\frac{22}{62} \rightarrow \frac{68,2}{118,2} \qquad \frac{\begin{array}{r}118,2\\-106,45\end{array}}{11,72}$$

$$KW_0 = 8805 = Ann \cdot 1,1^{-1} + Ann \cdot 1,1^{-2} + Ann \cdot 1,1^{-3}$$

$$Ann = KW_0 \cdot \frac{r \cdot (1+r)^n}{(1+r)^n - 1} = \frac{0,1 \cdot 1,1^3}{\underbrace{1,1^3 - 1}_{0,40211}} = 3541$$

$n \to \infty \Rightarrow Annf = r$

3a)

	t_1	t_2	t_3	t_4	t_5	t_6	...	
A:	40	40	40	40	40	40		r = 10%

$$KW_0 = \frac{Ann}{Annf} = \frac{Ann}{r} = \frac{40\,000}{0,1} = 400\,000$$

$$B: \quad 60 \quad 50 \quad \underbrace{70 \quad 70 \quad 70 \quad 70 \quad ...}_{} \qquad r = 10\%$$

$$\frac{+700}{750} \leftarrow \qquad \underset{(3 \to \infty)}{KW_2} = \frac{Ann}{r} = 700\,000$$

$$KW_0 = 60 \cdot 1,1^{-1} + 750 \cdot 1,1^{-2} = 679\,390$$

$$KW_0 = -A_0 + Q_1 (1+r)^{-1} + Q_2 (1-r)^{-2} + ... \qquad \boxed{?r?}$$

$$0 = -A_0 + Q_1 (1+p)^{-1} + Q_2 (1+p)^{-2}$$

$p > r \Rightarrow$ real

$p < r \Rightarrow$ alternative

38)

$KW_0 = \emptyset = -80 + 20 \cdot (1+p)^{-1} + 40 (1+p)^{-2} + 50 (1+p)^{-3}$

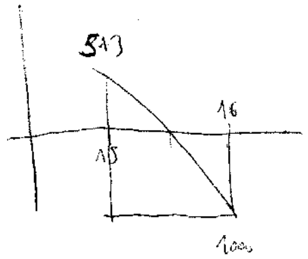
$p = 15\% \Rightarrow KW_0 = 513$

$p = 16\% \Rightarrow KW_0 = -1000$

$\Rightarrow p = 15{,}339\%$

$\frac{513+1000}{1} \approx \frac{513}{x}$ ähnliche Dreiecke

$x \approx 0{,}339$



6.5.

39.)

A : -70 25 30 40

B : -50 25 40

r=10%

$KW_A = 7575{,}25$

$KW_B = 5785{,}12$

$Ann = KW \cdot \frac{(1+r)^n \cdot r}{(1+r)^n - 1}$

$Ann_a = 7575{,}25 \cdot \frac{(1{,}1)^3 \cdot 0{,}1}{1{,}1^3 - 1} = 3045$

<

$Ann_b = 5785{,}12 \cdot \frac{1{,}1^2 \cdot 0{,}1}{1{,}1^2 - 1} = 3333$

	0	1	2	3	4	5	6
2*A:	-70	25	30	40 -70	25	30	40
3*B:	-50	25	40 -50	25	40 -50	25	40

$KW_{2A} = 7575{,}25 + 7575{,}25 \cdot 1{,}1^{-3} = 13263$

<

$KW_{3B} = 5785{,}12 + 5785{,}12 \cdot 1{,}1^{-2} + 5785{,}12 \cdot 1{,}1^{-4} = 14571{,}5$

$Ann_{2A} = 13263 \cdot \frac{1{,}1^6 \cdot 0{,}1}{1{,}1^6 - 1} = 3045$

$Ann_{3B} = 3333$

40)

$$\begin{array}{llllll} & & & & & KW \\ A: & -1000 & 900 & 1050 & \rightarrow & 686 \\ B: & -2000 & 1000 & 3150 & \rightarrow & 1512 \\ \hline B-A: & -1000 & 100 & 2100 & \rightarrow & 826 \end{array}$$

41)

$$\begin{array}{llllll} & & & & & KW \\ A: & -40 & 20 & 30 & \rightarrow & 2{,}973 \\ B: & -41 & 30 & 20 & \rightarrow & 2{,}802 \end{array}$$

$$A:\ 0 = -40 + 20\cdot(1+p)^{-1} + 30\cdot(1+p)^{-2} \quad \Big| \cdot (1+p)^2 \ : -40$$

$$0 = (1+p)^2 - \frac{1}{2}(1+p) - \frac{3}{4} \Rightarrow (1+p) = 1{,}1514 \Rightarrow p = 0{,}1514 \mathrel{\hat{=}} 15{,}14\,\%$$

$$B:\ \ p = 1{,}1543 \mathrel{\hat{=}} 15{,}43\,\%$$

$$\begin{array}{lllll} & 0 & 1 & 2 & 3 \\ A' & - & -40 & 20 & 30 \end{array} \qquad KW_{A'} = KW_A \cdot 1{,}1^{-1} = 2{,}704$$

$$A':\ 0 = ~~000~~ -40(1+p)^{-1} + 20(1+p)^{-2} + 30(1+p)^{-3} \quad \Big| \cdot (1+p)^3 : -40$$

$$0 = (1+p)^2 - \frac{1}{2}(1+p) - \frac{3}{4} \Leftrightarrow p = 15{,}14\,\%$$

interner Zinsfuß ist unabhängig von Zeitverschiebungen aber sehr sensibel auf unkomplette Zahlungsströme (⇒ nichts weglassen)

44.)

$r = 12\%$

							KW
A:	-250	100	100	100	70	→	34670
B:	-600	250	200	200	200	→	52113
B-A	-350	150	100	100	130	→	17443

$$KW_{Dif} = -350 + 150(1+p)^{-1} + 100(1+p)^{-2} + 100(1+p)^{-3} + 130(1+p)^{-4}$$

$p = 12\% \Rightarrow 17443$

$p = 15\% \Rightarrow -3871$

$p = 14\% \Rightarrow 2933$

$$\frac{2933 + 3871}{1} = \frac{2933}{x}$$

$x = 0{,}43 \Rightarrow p = 14{,}43\%$

13.5.

45.)

1,2 Mio. 5J. 8,5% $s = 30\%$

$$\text{Ann F} = \frac{r(1+r)^n}{(1+r)^n - 1} = \frac{0{,}085\,(1{,}085)^5}{1{,}085^5 - 1} = 0{,}25376$$

$$\text{Ann} = 1{,}2 \cdot 0{,}25376 = 304519$$

	0	1	2	3	4	5
Rückzahlungsrate		304519	304519	304519	304519	304519
Zinsen		102000	84786	66108	45844	23857
Tilgung		202519	219733	238411	258675	280662
Restschuld 1,2		997481	777748	539337	280662	0
Steuerersp.		30600	25436	19832	13753	7157

46.)

0	1	2	3	4	5
−100 000	–	–	–	–	+10 000
–	−40 000	−40 000	−40 000	−10 000	−10 000

$$-100000 + 10000(1+k)^{-5} = -40000\left((1+k)^{-1}+(1+k)^{-2}+(1+k)^{-3}\right) - 10000\left((1+k)^{-4}+(1+k)^{-5}\right)$$

$$-100000 + 40000\left((1+k)^{-1}+\ldots+(1+k)^{-3}\right) + 10000(1+k)^{-4} + 20000(1+k)^{-5} = 0$$

$k = 18{,}45\,\%$

Bei Zinssatz unter 18,45 ⇒ Kredit aufnehmen und kaufen

48

	s_1	s_2
a_1	20000	0
a_2	10000	10000

$E_1 = 0{,}5 \cdot 20\,000 + 0 \cdot 0{,}5 = 10\,000$

$E_2 = 10\,000$

a) $u(x) = x^{0,5}$

	s_1	s_2
a_1	$20000^{0,5}$	
a_2	$10000^{0,5}$	$10000^{0,5}$

$E_1(u(x)) = 0{,}5 \cdot 20000^{0,5} + 0 = 70{,}7$

$E_2(u(x)) = 10000^{0,5} = 100$

$70{,}7 = x^{0,5} \Rightarrow x = 5000$

b) $u(x) = x^2$

	s_1	s_2
a_1	20000^2	
a_2	10000^2	10000^2

$E_1(u(x)) = 0{,}5 \cdot 20000^2 = 200\,000\,000$

$E_2(u(x)) = 10\,000^2 = 100\,000\,000$

$200 \cdot 10^6 = x^2 \Rightarrow x = 14\,142$

49.) Laplace

$17{,}4 = 17 \cdot 0{,}2 + 2 \cdot 0{,}2 - 13 \cdot 0{,}2 + 30 \cdot 0{,}2 + 55 \cdot 0{,}2$

$19{,}4$

$\boxed{20{,}8}$

Savage-Niehans

S_1	S_2	S_3	S_4	S_5		
3	28	37	0	0	37	
3	20	9	5	25	$\boxed{25}$	⇒ kl. Fehler
0	0	0	8	47	47	

50.)

	s_1	s_2	s_3	Maximax	Hurwicz
a_1	49	9	25	49	$49 \cdot 0{,}1 + 9 \cdot 0{,}9 = 13$
a_2	4	64	81	81	11,7
a_3	16	100	1	100	10,9

52.) $E_1(u(x)) = \frac{1}{2} \cdot u(20000) + \frac{1}{2} \cdot u(40000) =$

$= \frac{1}{2} \cdot 36000 + \frac{1}{2} 64000 = 50000$

$E_2(u(x)) = \frac{1}{2} \cdot u(y) + \frac{1}{2} \cdot u(0) = 50000$

$\frac{1}{2} \cdot \left(\frac{y^2}{100000} + 2y \right) = 50000$

~~$y^2 + 100000y + 1 \cdot 10^{10} = 0$~~

$y^2 - 200000y + 100000^2 = 0$

$(y - 100000)^2 = 0$

$y = 100000$